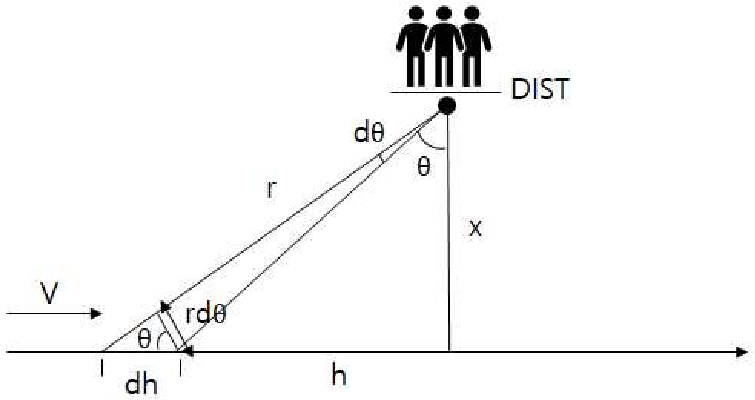
2.5.1 Dose to Persons Along the Transport Link While the Shipment is Moving

We derived the equation (19) in the technical manual, which is an expression for the total integrated dose to a stationary individual receptor at a perpendicular distance x from the path of a moving source with a dose rate factor k, passing a constant velocity V, as follows.



Initially it is noted that

$$\mathrm{dD} = \frac{\mathrm{k_0} \cdot \mathrm{DR_v} \cdot \mathrm{dt}}{\mathrm{r}^2}$$

and $\mathrm{dt} = \dfrac{\mathrm{dh}}{\mathrm{V}}$, $\quad \therefore \mathrm{dD} = \dfrac{\mathrm{k_0} \cdot \mathrm{DR_v}}{\mathrm{V}} \dfrac{\mathrm{dh}}{\mathrm{r}^2}$

Consider, $\mathrm{dh} = \mathrm{r}\sec\theta\,\mathrm{d}\theta$ and $\mathrm{r} = \mathrm{x}\sec\theta$, then $\mathrm{dr} = \mathrm{x}\sec\theta\tan\theta\,\mathrm{d}\theta = \sqrt{\mathrm{r}^2 - \mathrm{x}^2}\sec\theta\,\mathrm{d}\theta$

$$\therefore \mathrm{dD} = \frac{\mathrm{k_0} \cdot \mathrm{DR_v}}{\mathrm{V}} \frac{\mathrm{r\,dr}}{\mathrm{r}^2\sqrt{\mathrm{r}^2 - \mathrm{x}^2}} = \frac{\mathrm{k_0} \cdot \mathrm{DR_v}}{\mathrm{V}} \frac{\mathrm{dr}}{\mathrm{r}\sqrt{\mathrm{r}^2 - \mathrm{x}^2}}$$

Therefore, integrating r (from -∞ to ∞) instead of h, noting that this function is symmetric about the origin, results in the following expression for the total integrated dose to a stationary individual receptor.

$$\mathrm{D(x)} = \frac{2 \cdot \mathrm{k_0} \cdot \mathrm{DR_v}}{\mathrm{V}} \mathrm{I(x)}, \quad \mathrm{I(x)} = \int_{\mathrm{x}}^{\infty} \frac{e^{-\mu r}\mathrm{B(r)}}{\mathrm{r}\sqrt{(\mathrm{r}^2 - \mathrm{x}^2)}}\mathrm{dr}$$

where $\mathrm{D(x)}$ = total integrated dose absorbed by an individual at distance x (Sv)

$\mathrm{k_0}$ = point-source package shape factor (m^2)

$\mathrm{DR_v}$ = vehicle dose rate at 1 m from surface (mSv/hr)

V = vehicle speed (km/hr)

x = perpendicular distance of the individual receptor from shipment path (m)

μ = attenuation coefficient (m^{-1})

$B(r)$ = buildup factor expressed as a geometric progression for neutron

※ $B(r) = 1 + a_1 r + a_2 r^2 + a_3 r^3$ in RADTRAN

r = the distance between shipment and receptor along the route of travel (m)

When this general expression for $D(x)$ is multiplied by population density and integrated over the area of exposure, an expression for total population dose on both sides of the road is

$$D_{off} = PD \bullet DIST \bullet \int_{min}^{max} D(x)dx = \frac{4 \bullet k_0 \bullet DR_v}{V} \bullet PD \bullet DIST \bullet \int_{min}^{max} I(x)dx$$

where PD = population density (persons/km^2)

$DIST$ = a route or route segment distance (km)

In case of neutron, $I(x)$ is as below.

$$I(x) = \int_x^{\infty} \frac{e^{-\mu r} B(r)}{r\sqrt{(r^2 - x^2)}} dr = \int_x^{\infty} \frac{e^{-\mu r}(1 + a_1 r + a_2 r^2 + a_3 r^3)}{r\sqrt{(r^2 - x^2)}} dr$$

$$I(x) = \int_x^{\infty} \frac{e^{-\mu r}}{r\sqrt{(r^2 - x^2)}} dr + a_1 \int_x^{\infty} \frac{e^{-\mu r}}{\sqrt{(r^2 - x^2)}} dr + a_2 \int_x^{\infty} \frac{r e^{-\mu r}}{\sqrt{(r^2 - x^2)}} dr + a_3 \int_x^{\infty} \frac{r^2 e^{-\mu r}}{\sqrt{(r^2 - x^2)}} dr$$

Considering below modified Bessel function of the second kind1), we integrated $I(x)$ as follows.

$$K_\nu(z) = \frac{\sqrt{\pi}\,(\frac{1}{2})^\nu}{\Gamma(\nu + \frac{1}{2})} \int_1^{\infty} e^{-zt}(t^2 - 1)^{\nu - \frac{1}{2}} dt$$

$$K_0(z) = \int_1^{\infty} \frac{e^{-zt}}{\sqrt{t^2 - 1}} dt \quad \text{and} \quad K_1(z) = z\int_1^{\infty} e^{-zt}\sqrt{t^2 - 1}\, dt$$

Let $t = \frac{r}{x}$ then $dt = \frac{dr}{x}$

1) Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables, Abramowitz and Stegun (1964)

$$K_0(z) = \int_x^{\infty} \frac{e^{-\frac{z}{x}r}}{\sqrt{r^2 - x^2}}\,dr \quad \text{and} \quad K_1(z) = \frac{z}{x^2}\int_x^{\infty} e^{-\frac{z}{x}r}\sqrt{r^2 - x^2}\,dr$$

$$\therefore K_0(\mu x) = \int_x^{\infty} \frac{e^{-\mu r}}{\sqrt{r^2 - x^2}}\,dr \quad \text{and} \quad K_1(\mu x) = \frac{\mu}{x}\int_x^{\infty} e^{-\mu r}\sqrt{r^2 - x^2}\,dr$$

(1) the first term of $I(x)$

$$\int_x^{\infty} \frac{e^{-\mu r}}{r\sqrt{(r^2 - x^2)}}\,dr = \int_x^{\infty} \frac{e^{-\mu r}}{r}\frac{1}{\sqrt{(r^2 - x^2)}}\,dr = \int_{\mu}^{\infty} e^{-tr}\,dt \int_x^{\infty} \frac{1}{\sqrt{(r^2 - x^2)}}\,dr$$

$$= \int_{\mu}^{\infty}\int_x^{\infty} \frac{e^{-tr}}{\sqrt{(r^2 - x^2)}}\,dr\,dt = \int_{\mu}^{\infty} K_0(tx)\,dt$$

Let $tx = w$ then $dt = \dfrac{dw}{x}$

$$\therefore \int_x^{\infty} \frac{e^{-\mu r}}{r\sqrt{(r^2 - x^2)}}\,dr = \int_x^{\infty} K_0(tx)\,dt = \frac{1}{x}\int_{\mu x}^{\infty} K_0(w)\,dw$$

(2) the second term of $I(x)$

$$\int_x^{\infty} \frac{e^{-\mu r}}{\sqrt{(r^2 - x^2)}}\,dr = K_0(\mu x)$$

(3) the third term of $I(x)$

Consider $\dfrac{x}{\mu}K_1(\mu x) = \displaystyle\int_x^{\infty} e^{-\mu r}\sqrt{r^2 - x^2}\,dr$ and $\displaystyle\int u'v = uv - \int uv'$

Let $u' = e^{-\mu r}$, $v = \sqrt{r^2 - x^2}$ then, $u = -\dfrac{1}{\mu}e^{-\mu r}$, $v' = \dfrac{r}{\sqrt{r^2 - x^2}}$

$$\therefore \frac{x}{\mu}K_1(\mu x) = \int_x^{\infty} e^{-\mu r}\sqrt{r^2 - x^2}\,dr = \left[-\frac{1}{\mu}e^{-\mu r}\sqrt{r^2 - x^2}\right]_x^{\infty} + \frac{1}{\mu}\int_x^{\infty} \frac{r e^{-\mu r}}{\sqrt{r^2 - x^2}}\,dr$$

$$= \frac{1}{\mu}\int_x^{\infty} \frac{r e^{-\mu r}}{\sqrt{r^2 - x^2}}\,dr$$

$$\therefore \int_x^{\infty} \frac{r e^{-\mu r}}{\sqrt{r^2 - x^2}}\,dr = x K_1(\mu x)$$

(4) the fourth term of $\mathrm{I}(\mathrm{x})$

Consider

$$\frac{\mathrm{x}}{\mu}K_1(\mu\mathrm{x}) = \int_{\mathrm{x}}^{\infty} \mathrm{e}^{-\mu\mathrm{r}}\sqrt{\mathrm{r}^2-\mathrm{x}^2}\,\mathrm{dr} = \int_{\mathrm{x}}^{\infty} \frac{\mathrm{e}^{-\mu\mathrm{r}}}{\sqrt{\mathrm{r}^2-\mathrm{x}^2}}(\mathrm{r}^2-\mathrm{x}^2)\mathrm{dr}$$

$$= \int_{\mathrm{x}}^{\infty} \frac{\mathrm{r}^2\mathrm{e}^{-\mu\mathrm{r}}}{\sqrt{\mathrm{r}^2-\mathrm{x}^2}}\mathrm{dr} - \mathrm{x}^2\int_{\mathrm{x}}^{\infty} \frac{\mathrm{e}^{-\mu\mathrm{r}}}{\sqrt{\mathrm{r}^2-\mathrm{x}^2}}\mathrm{dr}\ (=\mathrm{x}^2K_0(\mu\mathrm{x}))$$

$$\therefore \int_{\mathrm{x}}^{\infty} \frac{\mathrm{r}^2\mathrm{e}^{-\mu\mathrm{r}}}{\sqrt{\mathrm{r}^2-\mathrm{x}^2}}\mathrm{dr} = \frac{\mathrm{x}}{\mu}K_1(\mu\mathrm{x}) + \mathrm{x}^2K_0(\mu\mathrm{x})$$

*Proof with Mathematica program (for $\mathrm{x}=1$ and $\mu=1$)

```
In[12]:= Integrate[Exp[-r] r^2/Sqrt[r^2 - 1], {r, 1, Infinity}]
         적분       지수 함수                          무한대

Out[12]= BesselK[0, 1] + BesselK[1, 1]
```

In conclusion,

$$\mathrm{I}(\mathrm{x}) = \int_{\mathrm{x}}^{\infty} \frac{\mathrm{e}^{-\mu\mathrm{r}}}{\mathrm{r}\sqrt{(\mathrm{r}^2-\mathrm{x}^2)}}\mathrm{dr} + \mathrm{a}_1\int_{\mathrm{x}}^{\infty} \frac{\mathrm{e}^{-\mu\mathrm{r}}}{\sqrt{(\mathrm{r}^2-\mathrm{x}^2)}}\mathrm{dr} + \mathrm{a}_2\int_{\mathrm{x}}^{\infty} \frac{\mathrm{r}\,\mathrm{e}^{-\mu\mathrm{r}}}{\sqrt{(\mathrm{r}^2-\mathrm{x}^2)}}\mathrm{dr} + \mathrm{a}_3\int_{\mathrm{x}}^{\infty} \frac{\mathrm{r}^2\mathrm{e}^{-\mu\mathrm{r}}}{\sqrt{(\mathrm{r}^2-\mathrm{x}^2)}}\mathrm{dr}$$

$$= \frac{1}{\mathrm{x}}\int_{\mu\mathrm{x}}^{\infty} K_0(\mathrm{w})\mathrm{dw} + \mathrm{a}_1K_0(\mu\mathrm{x}) + \mathrm{a}_2\mathrm{x}K_1(\mu\mathrm{x}) + \mathrm{a}_3\left[\frac{\mathrm{x}}{\mu}K_1(\mu\mathrm{x}) + \mathrm{x}^2K_0(\mu\mathrm{x})\right]$$

$$= \frac{1}{\mathrm{x}}\int_{\mu\mathrm{x}}^{\infty} K_0(\mathrm{w})\mathrm{dw} + (\mathrm{a}_1 + \mathrm{a}_3\mathrm{x}^2)K_0(\mu\mathrm{x}) + (\mathrm{a}_2\mathrm{x} + \mathrm{a}_3\frac{\mathrm{x}}{\mu})K_1(\mu\mathrm{x})$$

Contrary to above development, in the equation (23) of the technical manual of RADTRAN 6, the sign of the coefficient, $\mathrm{a}_3\mathrm{x}^2$ is negative and the whole expression is multiplied by $\mathrm{e}^{-\mu\mathrm{x}}$ as below.

$$\mathrm{I}(\mathrm{x}) = [\frac{1}{\mathrm{x}}\mathrm{BSKIN}_{\mu\mathrm{x}} + (\mathrm{a}_1 - \mathrm{a}_3\mathrm{x}^2)\mathrm{BESK0}_{\mu\mathrm{x}} + (\mathrm{a}_2\mathrm{x} + \mathrm{a}_3\frac{\mathrm{x}}{\mu})\mathrm{BESK1}_{\mu\mathrm{x}}] \cdot \mathrm{e}^{-\mu\mathrm{x}}$$

where $\mathrm{BSKIN}_{\mu\mathrm{x}}$ = Sandia National Lab's SLATEC math routine that computes repeated integrals of the modified Z-zero Bessel function for argument $(\mu\mathrm{x})$

$\mathrm{BESK0}_{\mu\mathrm{x}}$ = Sandia National Lab's SLATEC math routine that computes the modified Bessel function of the third kind of order zero for argument $(\mu\mathrm{x})$

$\mathrm{BESK1}_{\mu x}$ = Sandia National Lab's SLATEC math routine that computes the modified Bessel function of the third kind of order one for argument (μx)

However, we found that RADTRAN computes the equation below, not the equation (23) provided in the technical manual,

$$I(x) = \frac{1}{x}\mathrm{BSKIN}_{\mu x} + \left[(a_1 - a_3 x^2)\mathrm{BESK0}_{\mu x} + (a_2 x + a_3\frac{x}{\mu})\mathrm{BESK1}_{\mu x}\right] \bullet e^{-\mu x}$$

because when we compute $\int_{30}^{800} I(x)dx$ of above equation with MATLAB and compare it with $\int_{30}^{800} I(x)dx$ calculated with RADTRAN by changing the values of coefficients a_1, a_2 and a_3, it is confirmed that the maximum percent differences between two results are within 5 percent as shown in the below table.

RADTRAN†					MATLAB‡	Percent difference(%) $\frac{\lvert(1)-(2)\rvert}{(1)}\times 100$
μ	a_1	a_2	a_3	$\int_{30}^{800} I(x)dx^{(1)}$	$\int_{30}^{800} I(x)dx^{(2)}$	
7.42E-03	0	0	0	1.18E+00	1.16E+00	1.00
7.42E-03	1	0	0	6.44E+01	6.36E+01	1.25
7.42E-03	1	1	0	8.68E+03	8.62E+03	0.73
7.42E-03	1	1	1	5.28E+05	5.24E+05	0.75
7.42E-03	2.02E-02	6.17E-05	3.17E-08	3.00E+00	2.97E+00	1.16

†calculate Rural Off-Link dose - highway with Size = 3 m, DR_n = 1 mrem/hr, PD = 1 people/km^2, V = 1 km/hr, and the result is divided by 4 and 6.25 m^2(=k_0) and multiplied by (1000 mrem/rem)(1000 m/km)^2.

‡The integration is performed by use of a 8-point Gaussian-Legendre quadrature algorithm.

In conclusion, we believe that both there are necessary revisions to be considered in both the code algorithm and the technical manual.

2.5.2 Dose to Persons in Vehicles Sharing the Transport Link – Dose to Persons Traveling in the Same Direction

In the technical manual, two components of exposed population are considered: (1) people traveling in the same direction and (2) people immediately adjacent to the shipment in passing lanes.

1) Dose to Persons Traveling in the Same Direction – Highway Mode Only

In the technical manual, the shipment is modeled as moving at the same average speed V_v at the rest of traffic. Thus, vehicles traveling in the same direction as the shipment can be considered to be stationary with respect to the shipment and can be modeled as a linear continuum of vehicles beginning at some minimum distance beyond vehicle.

Therefore, the dose received by a person located at distance r from the shipment is computed by multiplying the dose rate by the duration of the total exposure time t

$$D = \frac{k_0 \cdot DR_v}{r^2} \cdot t, \text{ where } \quad t = \frac{DIST}{V_v}$$

The dose to persons traveling in the same direction as the shipment is

$$D_{same} = 2 \cdot k_0 \cdot DR_v \cdot \left(\frac{DIST}{V_v}\right) \cdot \left(\frac{N' \cdot PPV}{V_v}\right) \cdot \int_{min}^{\infty} \frac{e^{-\mu r} B(\mu r)}{r^2} dr$$

$$= \frac{2 \cdot k_0 \cdot DR_v \cdot DIST \cdot N' \cdot PPV}{V_v^2} \cdot Y(V_v)$$

where $Y(V_v) = Y_{V_v} = \int_{2V_v}^{\infty} \frac{e^{-\mu r} \cdot B(\mu r)}{r^2} dr$

N' = one-way traffic count (average number of vehicle per unit time)

PPV = average number of persons per vehicle

Here, the factor of 2 accounts for the fact that traffic is modeled as extended in two directions (both in front of and behind the shipment)

In case of neutron, Y_v is as below (hereinafter, let V_v = V).

$$Y_v = \int_{2v}^{\infty} \frac{e^{-\mu r} \cdot B(r)}{r^2} dr = \int_{2v}^{\infty} \frac{e^{-\mu r}(1 + a_1 r + a_2 r^2 + a_3 r^3 + a_4 r^4)}{r^2} dr$$

$$\mathrm{Y_v} = \int_{2v}^{\infty} \frac{e^{-\mu r}}{r^2} dr + a_1 \int_{2v}^{\infty} \frac{e^{-\mu r}}{r} dr + a_2 \int_{2v}^{\infty} e^{-\mu r} dr + a_3 \int_{2v}^{\infty} r e^{-\mu r} dr + a_4 \int_{2v}^{\infty} r^2 e^{-\mu r} dr$$

Considering the below exponential integral, we integrated $\mathrm{Y_v}$ as follows.

$$E_1(x) = \int_1^{\infty} \frac{e^{-tx}}{t} dt$$

Let $2Vt = r$ then $dt = \dfrac{dr}{2V}$ and $E_1(x) = \displaystyle\int_{2V}^{\infty} \frac{e^{-\frac{r}{2V}x}}{\frac{r}{2V}} \frac{dr}{2V} = \int_{2V}^{\infty} \frac{e^{-\frac{r}{2V}x}}{r} dr$

$$\therefore E_1(\mu \cdot 2V) = \int_{2V}^{\infty} \frac{e^{-\mu r}}{r} dr$$

(1) the first term of $\mathrm{Y_v}$

Consider $\displaystyle\int_{2V}^{\infty} \frac{e^{-\mu r}}{r^2} dr$ and $\displaystyle\int uv' = uv - \int u'v$

Let $u = e^{-\mu r}$, $v' = \dfrac{1}{r^2}$ then, $u' = -\mu e^{-\mu r}$, $v = -\dfrac{1}{r}$

$$\therefore \int_{2V}^{\infty} \frac{e^{-\mu r}}{r^2} dr = \left[-\frac{e^{-\mu r}}{r} \right]_{2V}^{\infty} - \mu \int_{2V}^{\infty} \frac{e^{-\mu r}}{r} dr = \frac{e^{-\mu \cdot 2V}}{2V} - \mu E_1(\mu \cdot 2V)$$

(2) the second term of $\mathrm{Y_v}$

$$\int_{2V}^{\infty} \frac{e^{-\mu r}}{r} dr = E_1(\mu \cdot 2V)$$

(3) the third term of $\mathrm{Y_v}$

$$\int_{2V}^{\infty} e^{-\mu r} dr = \left[-\frac{e^{-\mu r}}{\mu} \right]_{2V}^{\infty} = \frac{e^{\mu \cdot 2V}}{\mu}$$

(4) the fourth term of $\mathrm{Y_v}$

Consider $\displaystyle\int_{2V}^{\infty} r e^{-\mu r} dr$ and $\displaystyle\int uv' = uv - \int u'v$

Let $u = r$, $v' = e^{-\mu r}$ then, $u' = 1$, $v = -\dfrac{e^{-\mu r}}{\mu}$

$$\therefore \int_{2V}^{\infty} r e^{-\mu r} dr = \left[-\frac{r e^{-\mu r}}{\mu}\right]_{2V}^{\infty} + \frac{1}{\mu}\int_{2V}^{\infty} e^{-\mu r} dr = \frac{2V \cdot e^{-\mu \cdot 2V}}{\mu} + \frac{e^{-\mu \cdot 2V}}{\mu^2}$$

(5) the fifth term of Y_v

Consider $\int_{2V}^{\infty} r^2 e^{-\mu r} dr$ and $\int uv' = uv - \int u'v$

Let $u = r^2$, $v' = e^{-\mu r}$ then, $u' = 2r$, $v = -\frac{e^{-\mu r}}{\mu}$

$$\therefore \int_{2V}^{\infty} r^2 e^{-\mu r} dr = \left[-\frac{r^2 e^{-\mu r}}{\mu}\right]_{2V}^{\infty} + \frac{2}{\mu}\int_{2V}^{\infty} r e^{-\mu r} dr$$

$$= \frac{(2V)^2 \cdot e^{-\mu \cdot 2V}}{\mu} + \frac{2}{\mu}\left(\frac{2V \cdot e^{-\mu \cdot 2V}}{\mu} + \frac{e^{-\mu \cdot 2V}}{\mu^2}\right)$$

In conclusion,

$$Y_v = \int_{2v}^{\infty} \frac{e^{-\mu r}}{r^2} dr + a_1 \int_{2v}^{\infty} \frac{e^{-\mu r}}{r} dr + a_2 \int_{2v}^{\infty} e^{-\mu r} dr + a_3 \int_{2v}^{\infty} r e^{-\mu r} dr + a_4 \int_{2v}^{\infty} r^2 e^{-\mu r} dr$$

$$= \frac{e^{-\mu \cdot 2V}}{2V} - \mu E_1(\mu \cdot 2V) + a_1 E_1(\mu \cdot 2V) + a_2 \frac{e^{-\mu \cdot 2V}}{\mu} + a_3 \left(\frac{2V \cdot e^{-\mu \cdot 2V}}{\mu} + \frac{e^{-\mu \cdot 2V}}{\mu^2}\right)$$

$$+ a_4 \left(\frac{(2V)^2 \cdot e^{-\mu \cdot 2V}}{\mu} + \frac{2 \cdot (2V) \cdot e^{-\mu \cdot 2V}}{\mu^2} + \frac{2 e^{-\mu \cdot 2V}}{\mu^3}\right)$$

$$\therefore Y_v = (a_1 - \mu) E_1(\mu \cdot 2V) + e^{-\mu \cdot 2V}\left[\frac{1}{2V} + \left(\frac{a_2}{\mu} + \frac{a_3}{\mu^2} + \frac{2a_4}{\mu^3}\right) + (2V)\left(\frac{a_3}{\mu} + \frac{2a_4}{\mu^2}\right) + (2V)^2 \frac{a_4}{\mu}\right]$$

However, the equation (33) of the technical manual of RADTRAN 6 is different from above development as below.

$$Y_v = (a_1 - \mu) E_1(\mu \cdot 2V) + e^{-\mu \cdot 2V}[\frac{1}{2V} + \left(\frac{a_2}{\mu} + \frac{a_3}{\mu^2} + \frac{2a_4}{\mu^3}\right) + (2V)\left(\frac{a_3}{\mu^2} + \frac{2a_4}{\mu^3}\right) + (2V)^2 \frac{a_4}{\mu}]$$

2) Dose to Persons Immediately adjacent to the Shipment in Passing Lanes

In the technical manual, people immediately adjacent to the shipment are modeled identically to the others with the exception of minimum exposure distance. In addition, the traffic count is halved to equally distribute these people.

Therefore, recall

$$D_{same} = \frac{2 \cdot k_0 \cdot DR_v \cdot DIST \cdot N' \cdot PPV}{V^2} \cdot Y(V)$$

In case of the vehicles immediately adjacent to the shipment,

$$Y(V) = \frac{1}{2}\int_{x_{passing}}^{2V} \frac{1}{r^2} dr = \frac{1}{2}\left(\frac{1}{x_{passing}} - \frac{1}{2V}\right)$$

Here, the buildup factor could not be considered between $x_{passing} \leq r \leq 2V$.

Therefore, the dose received by persons in vehicles in the passing lane immediately adjacent to the shipment is

$$D(x) = \frac{2 \cdot k_0 \cdot DR_v \cdot DIST \cdot N' \cdot PPV}{V^2} \cdot \frac{1}{2}\left(\frac{1}{x_{passing}} - \frac{1}{2V}\right)$$

In conclusion, dose to persons traveling in the same direction is

$$D(x) = \frac{2 \cdot k_0 \cdot DR_v \cdot DIST \cdot N' \cdot PPV}{V^2} \cdot (F_1 + F_2)$$

where $F_1 = Y_v = \int_{2v}^{\infty} \frac{e^{-\mu r} \cdot B(\mu r)}{r^2} dr$ and $F_2 = \frac{1}{2}\left(\frac{1}{x_{passing}} - \frac{1}{2V}\right)$

However, the equation (35a) and (35b) of the technical manual of RADTRAN 6 are different from above development as below.

$$F_1 = 2Y_v V \text{ and } F_2 = \frac{1}{x} Y_v V$$

Based on the comparison of calculated results of test case using RADTRAN and MATLAB, we think that the RADTRAN computes equations which we derived, not the equations provided in the manual so it seems an editorial error in the manual. Therfore, we believe that there are necessary revisions to be considered in the technical manual.

*Calculation of test case for verification : Rural On-Link Dose for only neutron – Highway Mode

1) Input values

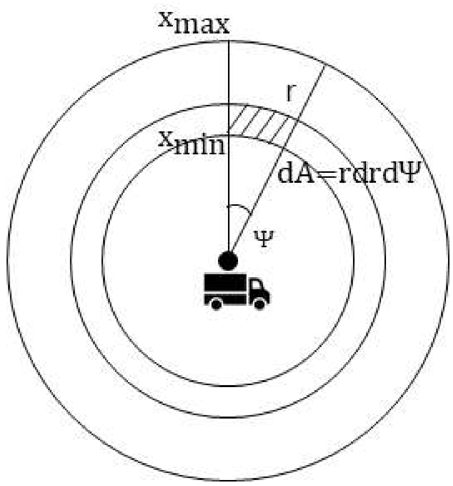
Source Term (neutron fraction=1, gamma fraction=0)	DR_n	14 mrem/hr
	μ	7.42E-03 m^{-1}
	a_1	2.02E-02
	a_2	6.17E-05
	a_3	3.17E-08
Vehicle	k_0	6.25 m^2
	V	88.49 km/hr
Population Density	DIST	1 km
	N'	470 vehicles/hr
	PPV	2 people/vehicle

2) Comparison of calculated results using MATLAB and RADTRAN

<table>
<tr><td colspan="3">1. Persons traveling in the opposite direction (MATLAB)</td></tr>
<tr><td>$I(x) = \int_x^\infty \frac{e^{-\mu r} B(r)}{r\sqrt{(r^2 - x^2)}} dr$</td><td colspan="2">$D_{\text{-opposite direction}}$</td></tr>
<tr><td>0.1299</td><td colspan="2">2.7285E-06</td></tr>
<tr><td colspan="3">2. Persons traveling in the same direction (MATLAB)</td></tr>
<tr><td>$Y_v = \int_{2v}^\infty \frac{e^{-\mu r} \cdot B(\mu r)}{r^2} dr$</td><td colspan="2">$D_{\text{-same direction}}$</td></tr>
<tr><td>0.0302</td><td colspan="2">6.3493E-07</td></tr>
<tr><td colspan="3">3. Persons in the vehicles passing the shipment (MATLAB)</td></tr>
<tr><td>$\frac{1}{2}\left(\frac{1}{x} - \frac{1}{2V}\right)$</td><td colspan="2">$D_{\text{-passing}}$</td></tr>
<tr><td>0.1148</td><td colspan="2">2.4123E-06</td></tr>
<tr><td rowspan="2">4. Total Dose</td><td>MATLAB</td><td>5.7757E-06</td></tr>
<tr><td>RADTRAN</td><td>5.66E-06</td></tr>
</table>

2.5.3 Dose to Population at Shipment Stops

With regard to option 2. Annular-Area Method,



$$D_{stop} = k_0 \cdot \frac{DR_v \cdot e^{-\mu r} \cdot B(\mu r)}{r^2} \cdot T_{st} \cdot ST_{st} \cdot \int PD_{st}\, dA$$

where k_0 = point-source package shape factor (m^2)

DR_v = vehicle dose rate at 1 m from surface (mSv/hr)

T_{st} = duration of stop (hr)

SF_{st} = shielding factor at stop

PD_{st} = population density of annular area at stop (persons/km^2)

Here, $dA = r\,dr\,d\Psi$

$$\therefore D_{stop} = k_0 \cdot DR_v \cdot T_{st} \cdot ST_{st} \cdot PD_{st} \cdot \int_0^{2\pi} d\Psi \int_{x_{min}}^{x_{max}} \frac{e^{-\mu r} \cdot B(\mu r)}{r^2}\, r\, dr$$

$$= 2\pi \cdot k_0 \cdot DR_v \cdot T_{st} \cdot ST_{st} \cdot PD_{st} \cdot \int_{x_{min}}^{x_{max}} \frac{e^{-\mu r} \cdot B(\mu r)}{r}\, dr$$

To solve

$$\int_{x_{min}}^{x_{max}} \frac{e^{-\mu r} \cdot B(r)}{r}\, dr = \int_{x_{min}}^{x_{max}} \frac{e^{-\mu r}(1 + a_1 r + a_2 r^2 + a_3 r^3 + a_4 r^4)}{r}\, dr$$

Let’s define

$$IR_s(x) = \int_x^{\infty} \frac{e^{-\mu r}}{r} + a_1 \int_x^{\infty} e^{-\mu r}\, dr + a_2 \int_x^{\infty} r\, e^{-\mu r}\, dr + a_3 \int_x^{\infty} r^2 e^{-\mu r}\, dr + a_4 \int_x^{\infty} r^3 e^{-\mu r}\, dr$$

(1) the first term

$$\int_{x}^{\infty} \frac{e^{-\mu r}}{r}\,dr = E_1(\mu x)$$

(2) the second term

$$\int_{x}^{\infty} e^{-\mu r}\,dr = \left[-\frac{e^{-\mu r}}{\mu}\right]_{x}^{\infty} = \frac{e^{-\mu x}}{\mu}$$

(3) the third term

Consider $\int_{x}^{\infty} r\,e^{-\mu r}\,dr$ and $\int uv' = uv - \int u'v$

let $u = r,\ v' = e^{-\mu r}$ then, $u' = 1,\ v = -\dfrac{e^{-\mu r}}{\mu}$

$$\therefore \int_{x}^{\infty} r\,e^{-\mu r}\,dr = \left[-\frac{r\,e^{-\mu r}}{\mu}\right]_{x}^{\infty} + \frac{1}{\mu}\int_{x}^{\infty} e^{-\mu r}\,dr = \frac{x\,e^{-\mu x}}{\mu} + \frac{e^{-\mu x}}{\mu^2}$$

(4) the fourth term

Consider $\int_{x}^{\infty} r^2 e^{-\mu r}\,dr$ and $\int uv' = uv - \int u'v$

Let $u = r^2,\ v' = e^{-\mu r}$ then, $u' = 2r,\ v = -\dfrac{e^{-\mu r}}{\mu}$

$$\therefore \int_{x}^{\infty} r^2 e^{-\mu r}\,dr = \left[-\frac{r^2 e^{-\mu r}}{\mu}\right]_{x}^{\infty} + \frac{2}{\mu}\int_{x}^{\infty} r\,e^{-\mu r}\,dr = \frac{x^2 e^{-\mu x}}{\mu} + \frac{2}{\mu}\left(\frac{x\,e^{-\mu x}}{\mu} + \frac{e^{-\mu x}}{\mu^2}\right)$$

(5) the fifth term

Consider $\int_{x}^{\infty} r^3 e^{-\mu r}\,dr$ and $\int uv' = uv - \int u'v$

Let $u = r^3,\ v' = e^{-\mu r}$ then, $u' = 3r^2,\ v = -\dfrac{e^{-\mu r}}{\mu}$

$$\therefore \int_{x}^{\infty} r^3 e^{-\mu r}\,dr = \left[-\frac{r^3 e^{-\mu r}}{\mu}\right]_{x}^{\infty} + \frac{3}{\mu}\int_{x}^{\infty} r^2 e^{-\mu r}\,dr$$

$$= \frac{x^3 e^{-\mu x}}{\mu} + \frac{3}{\mu}\left(\frac{x^2 e^{-\mu x}}{\mu} + \frac{2x\,e^{-\mu x}}{\mu^2} + \frac{2\,e^{-\mu x}}{\mu^3}\right)$$

Therefore,

$$IR_s(x) = \left[\frac{a_1}{\mu} + a_2\left(\frac{x}{\mu}+\frac{1}{\mu^2}\right) + a_3\left(\frac{x^2}{\mu}+\frac{2x}{\mu^2}+\frac{2}{\mu^3}\right) + a_4\left(\frac{x^3}{\mu}+\frac{3x^2}{\mu^2}+\frac{6x}{\mu^3}+\frac{6}{\mu^4}\right) \right] \cdot e^{-\mu x} + E_1(\mu x)$$

In conclusion,

$$\therefore D_{stop} = 2\pi \cdot k_0 \cdot DR_v \cdot T_{st} \cdot SF_{st} \cdot PD_{st} \cdot \int_{x_{min}}^{x_{max}} \frac{e^{-\mu r} \cdot B(\mu r)}{r} dr$$

$$= 2\pi \cdot k_0 \cdot DR_v \cdot T_{st} \cdot SF_{st} \cdot PD_{st} \cdot [IR_s(x_{min}) - IR_s(x_{max})]$$

$$\because \int_{x_{min}}^{x_{max}} = \int_{x_{min}}^{\infty} - \int_{x_{max}}^{\infty}$$

However, the equation (39) of the technical manual is different from above development as below.

$$IR_s(x) = \left[\frac{a_1}{\mu} + \boxed{\frac{a_2}{\mu}(\mu x+1)} + a_3\left(\frac{x}{\mu}+\frac{2x}{\mu^2}+\frac{2}{\mu^3}\right) + a_4\left(\frac{x^3}{\mu}+\frac{3x^2}{\mu^2}+\frac{6x}{\mu^3}+\frac{6}{\mu^4}\right) \right] \cdot e^{-\mu x} + E_1(\mu x)$$

Based on the comparison of calculated results of test case using RADTRAN and MATLAB, we think that the RADTRAN computes equations which we derived, not the equations provided in the manual so it seems an editorial error in the manual. Therfore, we believe that there are necessary revisions to be considered in the technical manual.

Input values	Source Term	the same as the above test case	
	Vehicle	k_0	6.25 m^2
		T_{st}	60 hr
	Population Density	SF_{st}	0.1
		PD_{st}	719 people/km^2
		Inner Radius	10 m
		Outer Radius	400 m
output	MATLAB	1.2922E-02 person-rem	
	RADTRAN	1.29E-02 person-rem	